3. UNIFORM FLOW AND ITS COMPUTATIONS

3.1 Introduction

A flow is said to be uniform if its properties remain constant with respect to distance. As mentioned in chapter one of the handout, the term open channel flow in open channel is to mean steady uniform flow. Since Q=AV, it follows that in uniform flow $V_1=V_2=V$. thus in uniform flow, depth of flow, area of cross section and velocity of flow remain constant along the channel. It is obvious; therefore, that uniform flow is possible only in ***prismatic (artificial) channels***.

3.2 Expressing the Velocity of a Uniform Flow

For hydraulic computation the mean velocity of a turbulent uniform flow in open channels is usually expressed approximately by a so-called uniform flow formula. Most practical uniform flow formula can be expressed in the following general form:

$$\boldsymbol{V = CR^x S^y} \quad \text{..(eqn.3.1)}$$

Where V is the mean velocity, R is the hydraulic gradient, S is the energy slope, x and y are exponents and C is a factor of flow resistance.

Chezy equation

By definition there is no acceleration in uniform flow. By applying the momentum equation to a control volume, distance L apart.

$$\sum F_x = W\ \sin\theta + F_1 - F_2 - F_3 - F_a = Q\rho(\beta_2 v_2 - v_1\beta_1) \quad \text{...........................(eqn. 3.2)}$$

Where $F_1$and F_2 are pressure forces and M_1 and M_2 are the momentum fluxes at section 1 and 2 respectively W weight of the fluid in the control volume and F_3 is the shear force at the boundary.

Since the flow is uniform:

$F_1=F_2$ and $M_1=M_2$

Also $W=\gamma AL$ and $F_3=\tau_O PL$

Where τ_O average shear stress on the wetted perimeter of length P and γ is unit weight of water. Replacing $\sin\theta$ by S_O(bottom slope) can be written as:-

$$\gamma ALS_o = \tau_o PL$$

$$\tau_o = \gamma \frac{A}{P} S_o = \gamma RS_o$$

Where $R = A/_P$ is defined as hydraulic radius.

Expressing the average shear stress τ_0 as $\tau_0 = K\rho V^2$, where k is a coefficient which depends on the nature of the surface and flow parameters.

$$\tau_o = \gamma \frac{A}{P} S_o = \gamma RS_o = k\rho V^2$$

Leading to is $V = C\sqrt{RS_0}$ known as *Chezy formula* Where $\mathbf{C} = \sqrt{\frac{\mathbf{g}}{\mathbf{k}}}$ a coefficient which depend on the nature of the surface and the flow and known as chezy coefficient.

Determination of Chezy resistance factor

Several forms of expressions for the Chezy coefficient C have been proposed by different investigations in the past. A few selected ones are selected below:

1. Pavlovski formula

$$C = \frac{1}{n} R^{x}$$

In which x= 2.5 $\sqrt{n} - 0.13 - 0.75\sqrt{R}\left(\sqrt{n} - 0.1\right)$ and n is Manning's coefficient

2. Ganguiller and Kutter formula

$$C = \frac{23 + \frac{1}{n} + \frac{0.00155}{S_o}}{1 + \left[23 + \frac{0.00155}{S_o}\right]\frac{n}{\sqrt{R}}}$$

3. Bazin's formula

$$C = \frac{87.0}{1 + M/R}$$ In which M is a coefficient dependant on the surface roughness

Manning's formula

A resistance formula proposed by Robert Manning, owing to its simplicity and acceptable degree of accuracy in a variety of practical applications, Manning's formula is probably the most widely used uniform flow formula in the world.

$$V = \frac{1}{n} R^{3/2} S_0^{1/2} \quad \text{………………………………..(eqn 3.3)}$$

Where n is roughness coefficient known as Manning's n [$L^{-1/3}$ T]. This coefficient is essentially a function of the nature of the boundary surface.

Comparing Chezy and Mannings:

$$C = \frac{1}{n} R^{1/6} \quad \text{…………………………..(eqn 3.4)}$$

In the Manning's formula, all the terms except n are capable of direct measurement. The roughness coefficient, being a parameter representing the integrated effect of the channel cross sectional resistance, is to be estimated. The selection of a value for n is subjective, based on one's experience and engineering judgments. However, a few aids are available which reduce to a certain extent the subjectiveness in the selection of an appropriate value of n for a given channel. These includes: A compressive list of various types of channels, there descriptions with the associated range of values of n and photographs of selected typical reaches of canals, there description and measured value of n.

Table 3. 1 Values of Roughness Coefficient *n*

Sl. No.	*Surface characteristics*	*Range of n*
(a)	*Lined channels with straight alignment*	
1	Concrete (a) formed, no finish	0.013-0.017
	(b) Trowel finish	0.011-0.015
	(c) Float finish	0.013-0.015
	(d) Gunite, good section	0.016-0.019
	(e) Gunite, wavy section	0.018-0.022
2	Concrete bottom, float finish, sides as indicated	
	(a) Dressed stone in mortar	0.015-0.017
	(b) Random stone in mortar	0.017-0.020
	(c) Cement rubble masonry	0.020-0.025
	(d) Cement-rubble masonry, plastered	0.016-0.020
	(e) Dry rubble (rip-rap)	0.020-0.030
3	Tile	0.016-0.018
4	Brick	0.0.14-0.017
5	Sewers (concrete, A.C., vitrified-clay pipes)	0.012-0.015
6	Asphalt (i) Smooth	0.013
	(ii) Rough	0.016
7	Concrete lined, excavated rock	
	(i) good section	0.017-0.020
	(ii) irregular section	0.022-0.027
8	Laboratory flumes-smooth metal bed and glass or perspex sides	0.009-0.010
(b)	*Unlined, non-erodible channels*	
1	Earth, straight and uniform	
	(i) clean, recently completed	0.016-0.020
	(ii) clean, after weathering	0.018-0.025
	(iii) gravel, uniform section, clean	0.022-0.030
	(iv) with short grass, few weeds	0.022-0.033
2	Channels with weeds and brush, uncut	
	(i) dense weeds, high as flow depth	0.05-0.12
	(ii) clean bottom, brush on sides	0.04-0.08
	(iii) dense weeds or aquatic plants in deep channels	0.03-0.035
	(iv) grass, some weeds	0.025-0.033
3	Rock	0.025-0.045
(c)	*Natural channels*	
1	Smooth natural earth channels, free from growth, little curvature	0.020
2	Earth channels, considerably covered with small growth	0.035
3	Mountain streams in clean loose cobbles, rivers with variable section with some vegetation on the banks	0.04-0.05
4	Rivers with fairly straight alignment, obstructed by small trees, very little under brush	0.06-0.075
5	Rivers with irregular alignment and cross-section, covered with growth of virgin timber and occasional patches of bushes and small trees	0.125

Factors affecting n

It is not common for engineers to think of a channel as having a single value of n for all occasions. In reality, the value of n is highly variable and depends on a number of factors. In selecting a proper value of n for various design conditions, a basic knowledge of these factors should be found useful.

The manning's n is essentially a coefficient representing the integrated effect of a large number of factors contributing to the energy loss in a reach. Some important factors are:

a) Surface roughness

b) Vegetation

c) Channel irregularity and

d) Channel alignment.

The chief among these are the characteristics of the surface.

- ***Surface roughness:*** represented by the size and shape of the grains of the material forming the weighted perimeter and producing a retarding effect on the flow. Fine grains result in relatively low value of n and course grains, in a high value of n.
- ***Vegetation:*** also markedly reduce the capacity of the channel and retard the flow.
- ***Channel irregularity:*** comprises irregularity in wetted perimeter and variation in cross-section and shape along the channel length.
- ***Channel alignment:*** smooth curves with larger radius will give a relatively low value of n, whereas sharp curvatures with severe meandering will increase n.

Empirical formula for n

Many empirical formulas have been presented for estimating Manning's coefficient n in natural streams. These relate n to the bed particle size. The most popular form under this type is the Strickler formula:

$$n = d_{50}^{1/6} \quad \text{.....(eqn.3.5)}$$

Where $\mathbf{d_{50}}$ is in meters represents the particle size in which 50% of the bed material is finer. For mixtures of bed material with considerable course grained size the above equation is modified by Mayer as:

$$n = \frac{d_{90}^{1/6}}{21.1} \quad \text{.....(eqn. 3.6)}$$

Where d_{90} is size in meters with 90% of the particle are finer than $\mathbf{d_{90}}$.

3.3 Composite roughness of channels

In some channels different parts of the channel perimeter may have different roughnesses. Canals, in which only the sides are lined, laboratory flumes with glass walls and rough beds, rivers with a sand bed in deep water portion and flood plains covered with vegetation, are some typical examples. For such channels it is necessary to determine an equivalent roughness, a coefficient that can be applied to the entire cross-sectional perimeter in using the Manning's formula. This equivalent roughness, also called the composite roughness, represents a weighted average value for the roughness coefficient. One such method of calculations of equivalent roughness is given below.

Consider a channel having its perimeter composed of N types of roughness P_1, P_2 , P_3… P_N are the lengths of these N parts and n_1, n_2 n_3 are the respective roughness coefficients. Let each part P_i be associated with a partial area A_i such that:

$$\sum_{i=1}^{N} A_i = A_1 + A_2 + A_3 \ldots A_N = A \qquad \text{Total area}$$

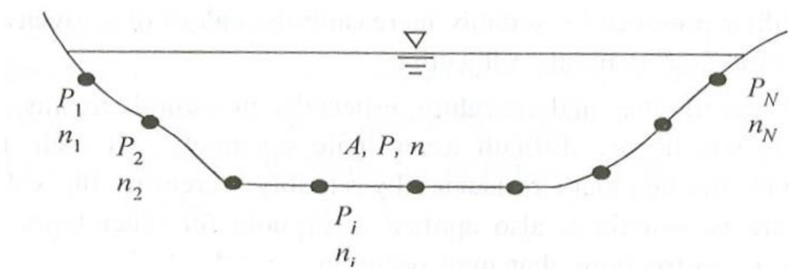


Fig 3.1 multi-roughness type section

It is assumed that the mean velocity in each partial area is the mean velocity V for the entire flow of the area, i.e.

$V_1 = V_2 = V_3 = \cdots Vi = V_N = V$

By manning's formula

$$S_0^{1/2} = \frac{V_1 n_1}{R_1^{2/3}} = \frac{V_2 n_2}{R_2^{2/3}} = \cdots = \frac{V_i n_i}{R_i^{2/3}} = \frac{V_N n_N}{R_N^{2/3}} = \frac{Vn}{R^{2/3}}$$ Where n is equivalent roughness.

$$\left(\frac{A_i}{A}\right)^{2/3} = \frac{n_i P_i^{2/3}}{n P^{2/3}}$$

$$A_i = A \frac{n_i^{3/2} P_i}{n^{3/2} P}$$

$$\sum A_i = A \frac{\sum \left(n_i^{3/2} P_i \right)}{n^{3/2} P}$$

$$n = \frac{\left(\sum n_i^{3/2} P_i \right)^{2/3}}{P^{2/3}}$$ ………………………………………………………..(eqn.3.7)

This equation affords a means of estimating the equivalent roughness of a channel having multiple roughness types in its perimeter.

3.4 uniform flow computation

The Manning's formula and the continuity equation Q=AV form the basic equations for uniform flow computations. The discharge Q is then given by:

$$Q = \frac{1}{n} A R^{2/3} S_o^{1/2}$$

Where, $K = \frac{1}{n} A R^{2/3}$ is called the conveyance of the channel and expresses the discharge capacity of the channel per unit longitudinal slope. The term $nK = AR^{2/3}$ is sometimes called the section factor for uniform flow computations. For a given channel, $AR^{2/3}$ is a function of depth of flow. $\frac{Qn}{S_o^{1/2}} = AR^{2/3}$ the write side of the equation contains the value of n, Q and S; but the left sides depends only on the geometry of the water area.

For example consider a trapezoidal channel with bottom width B and side slope m then:

$$A = (B + my)y$$

$$P = \left(B + 2y\sqrt{1+m^2}\right)$$

$$AR^{2/3} = \frac{(B+my)^{5/3} y^{5/3}}{\left(B + 2y\sqrt{m^2+1}\right)^{2/3}}$$

$$\frac{AR^{2/3}}{B^{8/3}} = \frac{Qn}{\sqrt{S_o} B^{8/3}} = \frac{(1+m\eta)^{5/3}\eta^{5/3}}{\left(1+2\eta\sqrt{m^2+1}\right)^{2/3}} \qquad \text{where } \eta = \frac{y}{B}$$

…………………………….. (eqn3.8)

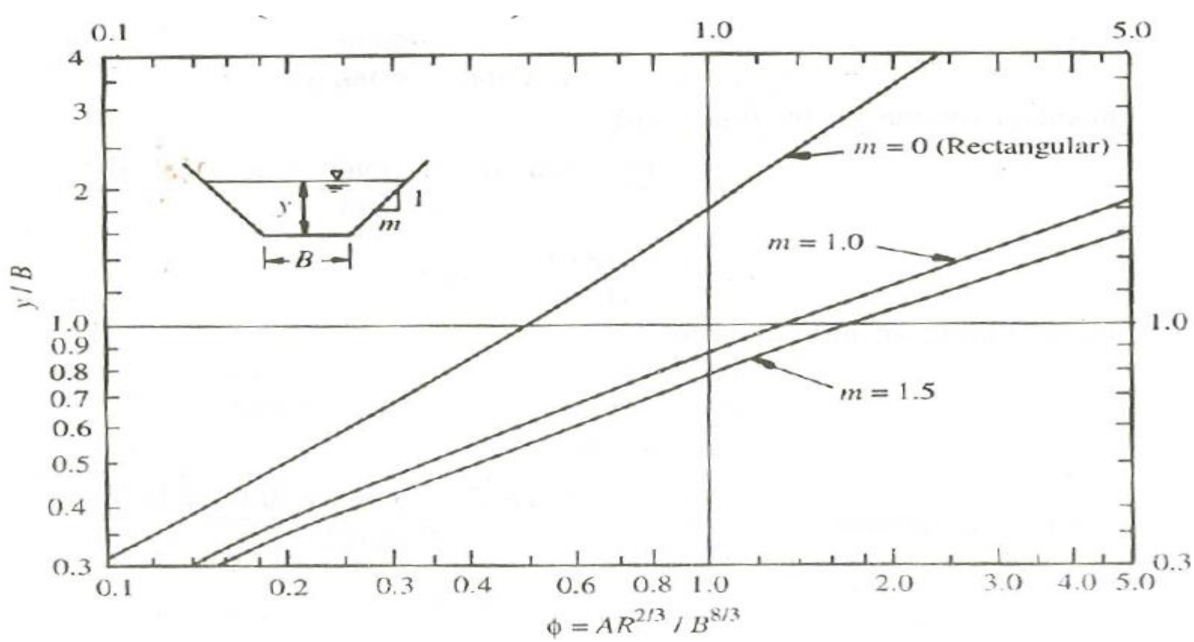


Figure 3.2 Variation of ϕ with y/B in trapezoidal channel

For a given channel, B and m are fixed and section factor is a function of depth only. The above figure shows the relationship of equation (3.8) in a non-dimensional manner by plotting $\emptyset = \frac{AR^{2/3}}{B^{8/3}}$ Vs y/B for different values of m. it may be seen that for m ≥ 0. There is only one value y/B for each value of ϕ, indicating that for m ≥ 0,$AR^{2/3}$ is a single valued function of y. this is true for any other shape of channels provided that the top width is either constant or increasing with depth these are called channels of the first kind.

Since $AR^{2/3} = \frac{Qn}{\sqrt{S_0}}$ and if n and S_0 are fixed for a channel, the channel of the first kind have unique depth in uniform flow associated with each discharge. This depth is called normal depth. Thus the normal depth is defined as the depth of flow at which a given discharge flows as uniform flow in a given channel. The normal depth is designated as y_0, the channels of the first kind have one normal depth only.

While a majority of the channels belong to the first kind, sometimes one encounters channels with closing top width. Circular and void sewers are typical examples of this category. Channels with a closing top-width can be designated as channels of the second kind.

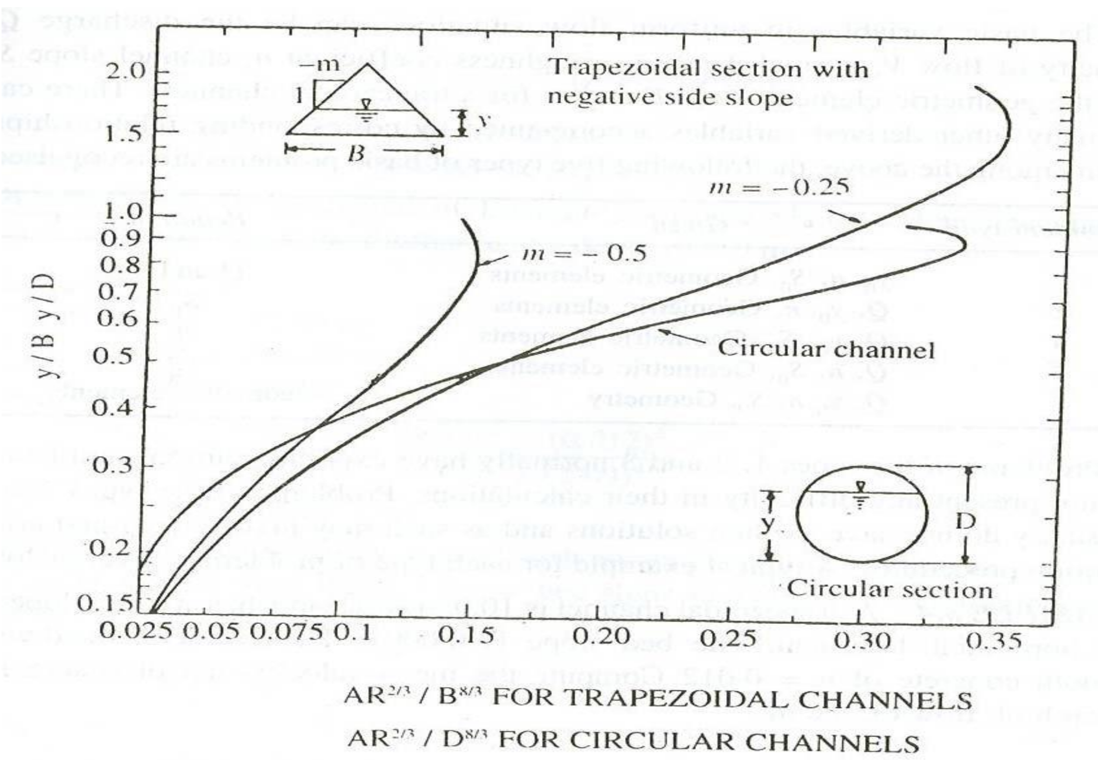


Figure 3.3 variation of $AR^{\frac{2}{3}}$ *in channels of the second kind*

The variation of $AR^{\frac{2}{3}}$ with depth of flow for a few channels of this second kind is shown in the above figure. It may be seen that in some ranges of depth, $AR^{\frac{2}{3}}$ is not a single valued function of depth.

Types problems in uniform flow

Uniform flow computation problems are relatively simple. The available relations are:

- Manning's formula
- Continuity equation
- Geometry of the cross-section

The basic variables in uniform flow situation can be the discharge Q, velocity of flow V, normal depth y_0, roughness coefficient n, channel slope S_0 and the geometric elements (e.g. *B* and *m* for trapezoidal channel). The following five types of basic problems are recognized in open channel flows.

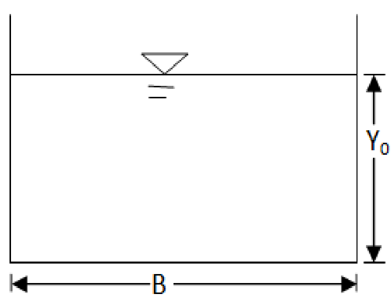
No	Given	Required
1	y_o, n, S_o, geometric elements	Q and V
2	y_o, n, Q, geometric elements	S_o
3	y_o, S_o, Q, geometric elements	n
4	n, S_o, Q, geometric elements	y_o
5	n, S_o, Q, y_o geometric	Geometric elements

Problems of type 1, 2 and 3 normally have explicit solutions and hence do not present any difficulty in their calculations. Problems of type 4 and 5 usually do not have explicit solutions and as such may involve trial-and-error solution procedure.

3.5 Computation of normal depth

It is evident from the above table that the calculation of the normal depth for many channels involves a trial-and-error solution. Since all open channel problems involve normal depth, special attention towards providing aids for quicker calculations of normal depth is warranted. A few aids for computing normal depth in some common channel sections are given below.

A).Wide Rectangular Channel With $y_0/B < 0.02$



For a rectangular channel, (Fig. 3.4)

Area $A = By_0$

Wetter perimeter $P = B + 2y_0$

Hydraulic radius

$$R = \frac{By_0}{B + 2y_0} = \frac{y_0}{1 + 2y_0/B}$$

Figure 3.4 rectangular channel

As $y0/B$, the aspect ratio of the channel decreases, $R \to y0$. Such channels with large bed-widths as compared to their respective depths are known as *wide rectangular channels*. In these channels, the hydraulic radius approximates to the depth of flow.

Considering a unit width of a wide rectangular channel,

$$A = y_0,\ R = y_0 \quad \text{and} \quad B = 1.0$$

$$\frac{Q}{B} = q = \text{discharge per unit width} = \frac{1}{n}\ y_0^{5/3}\ S_0^{1/2}$$

$$y_0 = \left[\frac{qn}{\sqrt{S_0}}\right]^{3/5} \quad \text{...} \quad (3.\ 9)$$

This approximation of a wide rectangular channel is found applicable to rectangular channels with $y_0/B < 0.02$.

B). Rectangular Channel With $y_0/_B \geq 0.02$

(b) Rectangular Channels with $y_0/B \geq 0.02$ For these channels $\frac{Qn}{\sqrt{S_0}} = AR^{2/3}$

$$AR^{2/3} = \frac{(By_0)^{5/3}}{(B+2y_0)^{2/3}} = \frac{(y_0/B)^{5/3}}{(1+2y_0/B)^{2/3}}\quad B^{8/3}$$

$$\frac{Qn}{\sqrt{S_0}\,B^{8/3}} = \frac{AR^{2/3}}{B^{8/3}} = \frac{(\eta_0)^{5/3}}{(1+2\eta_0)^{2/3}} = \phi(\eta_0) \qquad (3.10)$$

Where $\eta_0 = \frac{y_0}{B}$

Tables of $\phi\ (\eta_0)$ vs η_0 will provide a non-dimensional graphical solution aid for general application. Since $\phi = \frac{Qn}{\sqrt{S_0}\,B^{8/3}}$, one can easily find y_0/B from this table for any combination of Q, n, S_0 and B in a rectangular channel.

C.Trapezoidal Channel

Following a procedure similar to the above, for a trapezoidal section of side slope m: 1, (Fig. 3. 5)

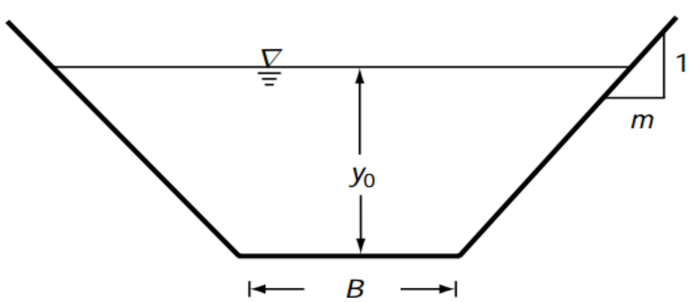


Figure 3.5trapezoidalChannel

Area $A = (B + my_0)\, y_0$

Wetter perimeter $P = (B + 2\sqrt{m^2 + 1}\, y_0)$

Hydraulic radius $R = A/P = \dfrac{(B + my_0)\, y_0}{(B + 2\sqrt{m^2+1}\, y_0)}$

$$\frac{Qn}{\sqrt{S_0}} = AR^{2/3} = \frac{(B + my_0)^{5/3}\, y_0^{5/3}}{(B + 2\sqrt{m^2+1}\, y_0)^{2/3}}$$

Non-dimensionalising the variables,

$$\frac{AR^{2/3}}{B^{8/3}} = \frac{Qn}{\sqrt{S_0}\, B^{8/3}} = \frac{(1 + m\eta_0)^{5/3}(\eta_0)^{5/3}}{(1 + 2\sqrt{m^2+1}\, \eta_0)^{2/3}} = \phi\,(\eta_0, m) \quad \text{.................(3.11)}$$

Where $\eta_0 = y_0/B$

Equation 3.11 could be represented as curves or Tables of ϕ vs η_0 with m as the third parameter to provide a general normal depth solution aid. It may be noted that $m = 0$ is the case of a rectangular channel. Table 3A.1 given in Appendix 3A at the end of this chapter gives values of ϕ for η_0 in the range 0.1 to 1.70 and m in the range 0 to 2.5. Values of η_0 are close enough for linear interpolation between successive values. This table will be useful in quick solution of a variety of uniform flow problems in rectangular and trapezoidal channels. Similar table of ϕ vs η_0 for any desired m values and ranges of η_0 can be prepared very easily by using a spread sheet such as MS Excel.

D. Circular Channel

Let D be diameter of a circular channel (Fig 3.6) and 2θ be the angle in radians subtended by the water surface at the centre.

A = area of the flow section

= area of the sector OMN – area of the triangle OMN

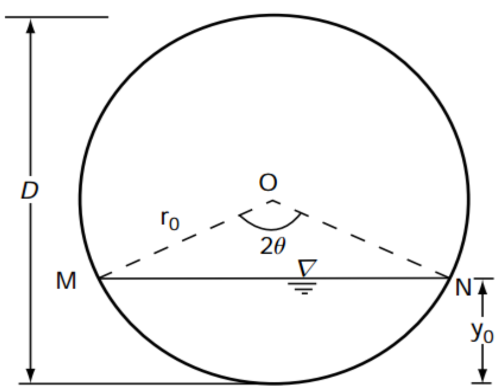


Fig. 3. 6 *Circular channel*

$$A = \frac{1}{2}\, r_0^2\, 2\theta - \frac{1}{2}.2r_0 \sin\theta.\, r_0 \cos\theta$$

$$= \frac{1}{2}\,(r_0^2\, 2\theta - r_0^2 \sin 2\theta)$$

$$= \frac{D^2}{8}\,(2\theta - \sin 2\theta)$$

P = wetted perimeter

$= 2r_0\,\theta = D\theta$

Also $\cos\theta = \dfrac{r_0 - y_0}{r_0} = \left(1 - \dfrac{2y_0}{D}\right)$

Hence $\theta = f(y_0/D)$

$$Q = \frac{1}{n} A R^{2/3} S_0^{1/2}$$

Assuming n = constant for all depths

$$\frac{Qn}{\sqrt{S_0}} = \frac{A^{5/3}}{P^{2/3}} = \frac{D^{10/3}}{8^{5/3}} \; \frac{(2\theta - \sin 2\theta)^{5/3}}{(D_\theta)^{2/3}}$$

Non-dimensionalising both sides

$$\frac{Qn}{\sqrt{S_0}\, D^{8/3}} = \frac{AR^{2/3}}{D^{8/3}} = \frac{1}{32} \; \frac{(\theta - \sin 2\theta)^{5/3}}{\theta^{2/3}}$$

$$= \phi\,(y_0/D) \qquad (3.12)$$

The functional relationship of Eq. 3.12 has been evaluated for various values of y_0/D and is given in Table 2A.1 in Appendix 2A. Besides $AR^{\frac{2}{3}}/D^{\frac{8}{3}}$, other geometric elements of a circular channel are also given in the table which is very handy in solving problems related to circular channels. Using this table, with linear interpolations wherever necessary, the normal depth for a given D, Q, n and $S0$ in a circular channel can be determined easily. The graphical plot of Eq. 3.12 is also shown in Fig. 3.3. As noted earlier, for depths of flow greater than 0.82D, there will be two normal depths in a circular channel. In practice, it is usual to restrict the depth of flow to a value of 0.8 D to avoid the region of two normal depths. In the region y/D > 0.82, a small disturbance in the water surface may lead the water surface to seek alternate normal depths, thus contributing to the instability of the water surface.

3.6 Hydraulically efficient channel section

The conveyance of a channel section of a given area increases with a decrease in its perimeter. Hence a channel section having the minimum perimeter for a given area of flow provides the maximum value of the conveyance. With the slope, roughness coefficients and area of flow fixed, a minimum perimeter section will represent the hydraulically efficient section as it conveys the maximum discharge. The channel section is also called the best section. Of all the various possible open channel sections, the semicircular shape has the least amount of perimeter of a given area.

(a) Rectangular Section Bottom width $= B$ and depth of flow $= y$

Area of flow $A = By =$ constant

Wetted perimeter $P = B + 2y$

$$= \frac{A}{y} + 2y$$

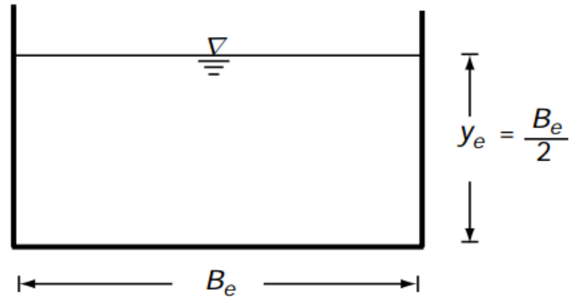


Fig. 3. 7 *Hydraulically efficient rectangular channel*

If P is to be minimum with A = constant,

$$\frac{dP}{dy} = -\frac{A}{y^2} + 2 = 0$$

Which gives $A = 2\,y_e^2$

i.e. $y_e = B_e/2,\ B_e = 2y_e$ and $R_e = \frac{y_e}{2}$ .... (3.13)

The suffix '*e*' denotes the geometric elements of a hydraulically efficient section. Thus it is seen that for a rectangular channel when the depth of flow is equal to half the bottom width, i.e., when the channel section is a half-square, a hydraulically efficient section is obtained (Fig. 3.7).

(b) Trapezoidal Section Bottom width $= B$, side slope $= m$ horizontal: 1 vertical

Area $A = (B + my)\,y =$ constant

$$B = \frac{A}{y} - my$$

Wetter perimeter $B + 2\,y\sqrt{m^2+1}$

$$= \frac{A}{y} - my + 2\,y\sqrt{m^2+1}$$

Keeping A and m as fixed, for a hydraulically efficient section,

$$\frac{dP}{dy} = -\frac{A}{y^2} - m + 2\sqrt{m^2+1} = 0$$

i.e. $A = \left(2\sqrt{1+m^2} - m\right) y_e^2$

Substituting and re arranging the above equations gives

$$B_e = 2\,y_e\left(\sqrt{1+m^2} - m\right)$$

$$P_e = 2\,y_e\left(2\sqrt{1+m^2} - m\right)$$

$$R_e = \frac{\left(2\sqrt{1+m^2} - m\right) y_e^2}{2\left(2\sqrt{1+m^2} - m\right) y_e} = y_e / 2$$ ....(3.14)

A hydraulically efficient trapezoidal section having the proportions given by Eqs 3.14 to is indicated in Fig. 3.8. Let *O* be center of the water surface. *OS* and *OT* are perpendiculars drawn to the bed and sides respectively.

$$OS = y_e$$

$$OT = OR \sin\theta = \frac{OR}{\sqrt{m^2+1}}$$

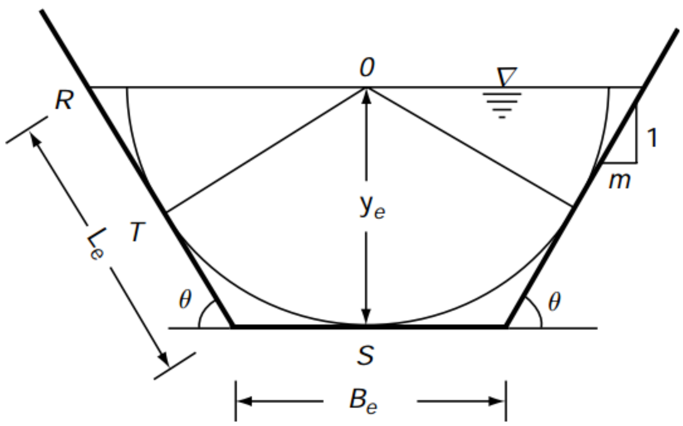


Fig. 3. 8 *Hydraulically efficient trapezoidal channel*

$$\text{OR} = \frac{1}{2}B_e + my_e.$$

Substituting for B_e from

$$OR = y_e\sqrt{1+m^2}$$

$$OT = OS = y_e$$

Thus the proportions of a hydraulically efficient trapezoidal section will be such that a semicircle can be inscribed in it.

In the above analysis, the side slope *m* was held constant. However, if *m* is allowed to vary, the optimum value of *m* to make *Pe most efficient* is obtained by putting $\frac{dP_e}{dm} = 0,$

$$P_e = 2\sqrt{A(2\sqrt{1+m^2} - m)}$$

Setting $\frac{dp_e}{dm} = 0$ in

$$m_{em} = \frac{1}{\sqrt{3}} = \cot\theta$$

$$\theta_{em} = 60^o$$

Where the suffix '*em*' denotes the most efficient section, Further.

$$p_{em} = 2y_{em}\left(2\sqrt{1+1/3} - \frac{1}{\sqrt{3}}\right) = 2\sqrt{3}\,y_{em}$$

$$B_{em} = 2y_{em}\left(2\sqrt{1+1/3} - \frac{1}{\sqrt{3}}\right) = \frac{2}{\sqrt{3}}y_{em}$$

$$A = \left(2\sqrt{1+1/3} - \frac{1}{\sqrt{3}}\right)y_{em}^2 = \sqrt{3}\,y_{em}^2$$

If L = length of the inclined side of the canal, it is easily seen that

$$L_{em} = \frac{2}{\sqrt{3}} y_{em} = B_{em}$$

Thus the hydraulically most efficient trapezoidal section is one-half of a regular hexagon.

Using the above approach, the relationship between the various geometrical elements to make different channel shapes hydraulically efficient can be determined. Table 3.4 contains the geometrical relation of some most efficient sections.

Table 3.4 *Proportions of Some Most Efficient Sections*

Sl. No	Channel Shape	Area (A_{em})	Wetted Perimeter (P_{em})	Width (B_{em})	Hydraulic Radius (R_{em})	Top width (T_{em})	$\frac{Qn}{y_{em}^{8/3} S_0^{1/2}} = K_{em}$
1	Rectangle (Half square)	$2\ y_{em}^2$	$4\ y_{em}$	$2\ y_{em}$	$\frac{y_{em}}{2}$	$2\ y_{em}$	1.260
2	Trapezoidal (Half regular hexagon, $m = \frac{1}{\sqrt{3}}$)	$\sqrt{3} y_{em}^2$	$2\sqrt{3} y_{em}$	$\frac{2}{\sqrt{3}} y_{em}$	$\frac{y_{em}}{2}$	$\frac{4\ y_{em}}{\sqrt{3}}$	1.091
3	Circular (semi-circular)	$\frac{\pi}{2}\ y^2_{em}$	πy_{em}	$D = 2y_{em}$	$\frac{y_{em}}{2}$	$2\ y_{em}$	0.9895
4	Triangle (Vertex angle = 90°)	y_{em}^2	$2\sqrt{3} y_{em}$	–	$\frac{y_{em}}{2\sqrt{2}}$	$2\ y_{em}$	0.500

3.7 Compound sections

Some channel sections may be formed as a combination of elementary sections. Typical natural Channels, such as rivers, have flood plains which are wide and shallow compared to the deep main Channel. The figure below represents a simplified section of a stream with flood banks. Channel of This kind is known as compound sections.

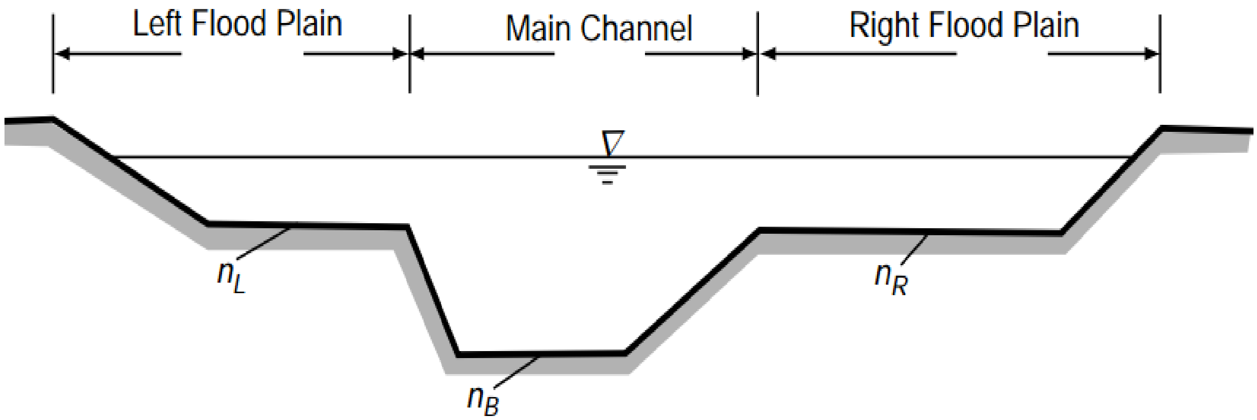


Figure 3.8a compound section

Consider the compound section to be divided into subsections by arbitrary lines. These can be either extensions for the deep channel boundaries as in the figure above or vertical lines drawn at the edge of the deep channels.

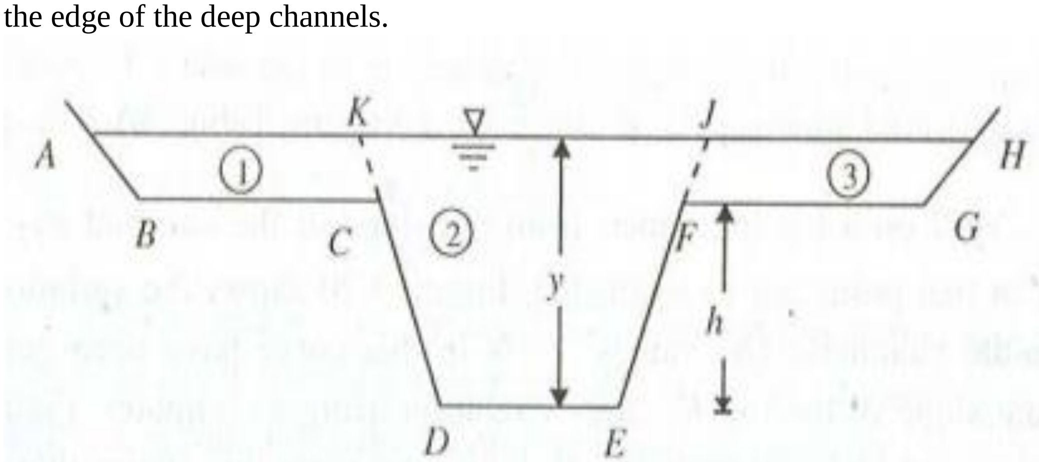


Figure 3.8b compound section

Assuming the longitudinal slope to be the same for all subsections, it is easy to see that the subsections will have different mean velocities depending up on the depth and roughness of the boundaries. Generally, overbanks have larger size roughness than the deeper main Channel. If the mean velocities **Vi** in the various subsections are known then the total discharge is:-

$$\sum V_i A_i$$

If the depth of flow is confined to the deep channel only (i.e y < h), calculation of discharge by using the Manning's is very simple. However, when the flow spills over into the flood plain (i.e. y>h), the problem of the discharge calculation is complicated as the calculation may give a smaller hydraulic radius for the whole stream section and hence the discharge may be under estimated.

This underestimation of discharge happens in all ranges in small ranges y, say h < y < y_m, where y_m is maximum value of y beyond which the underestimation of the discharge as above does occur. For the value of y > y_m, the calculation of the discharge by considering the whole section as one unit would be adequate. For the value of y in the range of h < y < y_m, the channel has to be considered to be made up of sub-areas and the discharge in each sub-area determined separately. The total discharge is obtained as a sum of discharge through all such sub-areas. The value of y_m would depend upon the channel geometry. However, for practical purpose the following method of discharge estimation can be adopted.

i.) The discharge is calculated as the sum of the partial discharge in the sub-areas; as 1, 2 and 3 as the above figure.

$$Q_p = \sum Q_i = \sum V_i A_i$$

ii.) The discharge is also calculated by considering the whole section as one unit, (Portion ABCDEFGH as one unit) say Q_w.

iii.) The larger of the above two discharges Qp and Qw, is adopted as the discharge at the depth y. For determining the partial discharge Qi and hence Qp in step one above, two methods are available.

1. ***Posey's method***:- in this method while calculating the wetted perimeter for the sub-areas, the imaginary divisions (FJ and CD refer the above figure) are considered as boundaries for the deeper sections only and neglected completely in the calculation relating to the shallower portion.
2. ***Zero shear method:***-some investigators mostly in computational work treat the interface as purely a hypothetical interface with zero shear stress. As such, the interfaces are not counted as perimeter either for the deep portion or for the shallow portion.

3.8 Generalized-Flow Relation

Since the Froude number of the flow in a channel is $F = \frac{V}{\sqrt{gA/T}} \longrightarrow \frac{Q^2}{g} = \frac{F^2A^3}{T}$

If the discharge Q occurs as a uniform flow, the slope S_0 required sustaining this discharge is, by the Manning's formula, $S_0 = \frac{Q^2n^2}{A^2R^{4/3}}$ Substituting in the above equation and simplifying

$$S_0 = \frac{F^2gn^2P^{4/3}}{TA^{1/3}} \quad \textbf{Or} \quad \frac{S_0}{F^2gn^2} = \frac{P^{4/3}}{TA^{1/3}} = f(y)$$

For a trapezoidal channel of side slope ***m*** $\frac{S_0}{F^2gn^2} = \frac{\left(B+2\sqrt{m^2+1}y\right)^{4/3}}{(B+2my_0)[(B+my)y_0]^{1/3}}$

Non-dimensionalising both sides, through multiplication

by $B^{1/3}$

$$\left(\frac{S_0B^{1/3}}{F^2gn^2}\right) = \frac{\left(1+2\sqrt{m^2+1}\,\eta\right)^{4/3}}{(1+2m\eta)(1+m\eta)^{1/3}} \quad \ldots\ldots\ldots\ldots\ldots(3.60)$$

in which $\eta = y_0/B$. Designating $\left(\frac{S_0B^{1/3}}{F^2gn^2}\right) = S_*$ = generalised slope $S_* = f(m,\eta)$

Equation (3.60) represents the relationship between the various elements of uniform flow in a trapezoidal channel in a generalized manner. The functional relationship of Eq. (3.60) is plotted in Fig. 3.9. This figure can be used to find, for a given trapezoidal channel, (a) the bed slope required to carry a uniform flow at a known depth and Froude number and (b) the depth of flow necessary for generating a uniform flow of a given Froude number in a channel of known bed slope.

For a rectangular channel, m =0 and hence Eq. (3.60) becomes $S_* = \frac{(1+2\eta)^{4/3}}{\eta^{1/3}}$ ………..(3.60a)

For a triangular channel, B = 0 and hence Eq. (3.60) cannot be used. However, by redefining the generalized slope for triangular channels, by Eq. (3.60a).

$$\frac{S_0 y^{1/3}}{F^2 g n^2} = S_{*t} = \left(2^{1/3}\right)\left(\frac{1+m^2}{m^2}\right) \quad \text{……………………………………(3.63)}$$

Roots and Limit Values of S. for Trapezoidal Channels

Equation (3.60) can be written as $S_*^3 = \frac{\left(1+2\eta\sqrt{1+m^2}\right)^4}{(1+2m\eta)^3\eta(1+m\eta)}$…………(3.64)

This is a fifth-degree equation in η, except for *m = 0* when it reduces to a fourth, degree equation. Out of its five roots it can be shown that

(a) At least one root shall be real and positive and

(b) Two roots are always imaginary.

Thus depending upon the value of m and S_*, there may be one, two or three roots.

The limiting values of S_* are obtained by putting, $\frac{dS_*}{d\eta}=0$, which results in

$$8\eta\sqrt{1+m^2}\,(1+\eta m)(1+2m\eta) - \left(1+2\eta\sqrt{1+m^2}\right)$$
$$\left(1+10m\eta+10m^2\eta^2\right) = 0 \quad \text{……………………………………(3.65)}$$

Solving Eq. (3.65) the following significant results are obtained

1. Rectangular channels (m=0), a single limiting value with S_* = 8/3 and η = l/6 is obtained.
2. Between m = 0 and m = 0.46635 there are two limiting values.
3. At m = 0.46635, the two limit values merge into one at S_*= 2.l545 and η =0.7849.
4. For m > 0.46635, there are no limiting points.

These features are easily discernible from Fig. 3.9

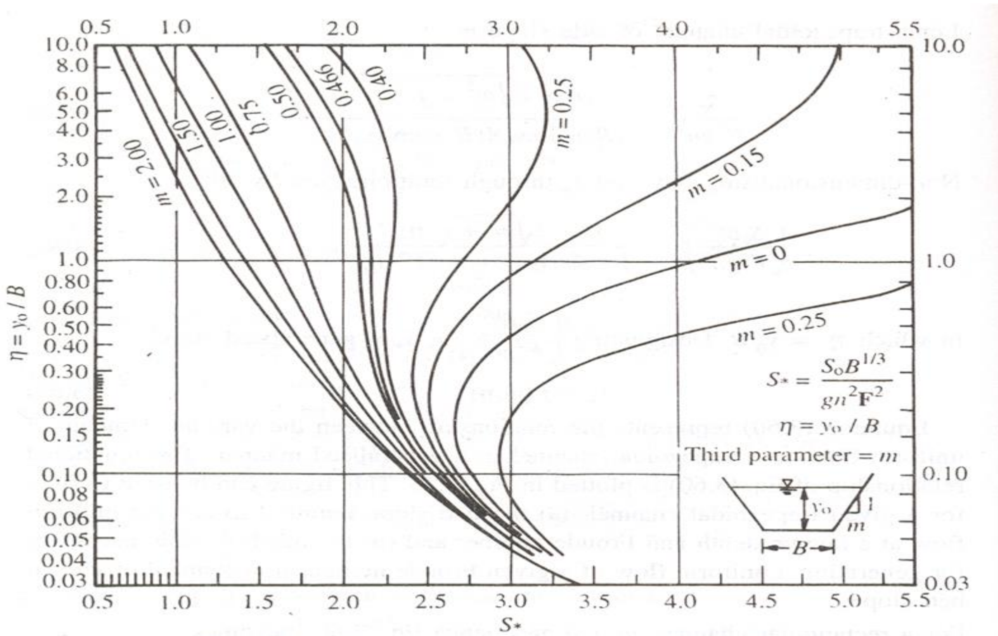


Fig. 3.9 Generalised flow relation [Ref. 25]

APPENDIX 3A

Table 3A.1 gives the variation of $\phi = f(\eta_0, m)$ as represented by Eq. (3.32) and provides a convenient aid to determine the normal depth in rectangular and trapezoidal channels. At the normal depth

$$\phi = \frac{nQ}{\sqrt{S_0}\, B^{8/3}} \quad \text{and} \quad \eta_0 = \frac{y_0}{B}.$$

Note that the column $m = 0$ corresponds to a rectangular channel

Table 3A-1 *Values of ϕ for Trapezoidal Channels*

η_0	Value of ϕ					η_0	Value of ϕ				
	m = 0	m = 1.0	m = 1.5	m = 2.0	m = 2.5		m = 0	m = 1.0	m = 1.5	m = 2.0	m = 2.5
0.100	0.01908	0.02139	0.02215	0.02282	0.02345	0.155	0.03736	0.04463	0.04713	0.04938	0.05151
0.105	0.02058	0.02321	0.02407	0.02484	0.02556	0.160	0.03919	0.04708	0.04982	0.05227	0.05459
0.110	0.02212	0.02509	0.02607	0.02694	0.02776	0.165	0.04104	0.04960	0.05257	0.05523	0.05777
0.115	0.02369	0.02702	0.02814	0.02912	0.03005	0.170	0.04292	0.05217	0.05539	0.05828	0.06104
0.120	0.02529	0.02902	0.03027	0.03138	0.03243	0.175	0.04482	0.05479	0.05828	0.06141	0.06439
0.125	0.02693	0.03108	0.03247	0.03371	0.03489	0.180	0.04675	0.05747	0.06123	0.06462	0.06785
0.130	0.02860	0.03319	0.03475	0.03613	0.03744	0.185	0.04869	0.06021	0.06426	0.06791	0.07139
0.135	0.03029	0.03537	0.03709	0.03862	0.04007	0.190	0.05066	0.06300	0.06735	0.07128	0.07503
0.140	0.03202	0.03760	0.03949	0.04119	0.04280	0.195	0.05265	0.06584	0.07052	0.07474	0.07876
0.145	0.03377	0.03988	0.04197	0.04384	0.04561	0.200	0.05466	0.06874	0.07375	0.07827	0.08259
0.150	0.03555	0.04223	0.04452	0.04657	0.04852						

Table 3A-1 *(Continued)*

η_o	Value of ϕ					η_o	Value of ϕ				
	m = 0	m = 1.0	m = 1.5	m = 2.0	m = 2.5		m = 0	m = 1.0	m = 1.5	m = 2.0	m = 2.5
0.205	0.05668	0.07170	0.07705	0.08189	0.08651	0.455	0.17485	0.28970	0.33534	0.37756	0.41811
0.210	0.05873	0.07471	0.08042	0.08559	0.09053	0.460	0.17744	0.29550	0.34249	0.38597	0.42774
0.215	0.06079	0.07777	0.08386	0.08938	0.09464	0.465	0.18005	0.30136	0.34972	0.39448	0.43749
0.220	0.06287	0.08089	0.08737	0.09325	0.09885	0.470	0.18265	0.30728	0.35703	0.40310	0.44736
0.225	0.06497	0.08406	0.09095	0.09720	0.10316	0.475	0.18527	0.31326	0.36443	0.41183	0.45737
0.230	0.06709	0.08729	0.09460	0.10124	0.10757	0.480	0.18789	0.31929	0.37191	0.42066	0.46751
0.235	0.06922	0.09057	0.09832	0.10536	0.11208	0.485	0.19051	0.32538	0.37947	0.42960	0.47778
0.240	0.07137	0.09391	0.10211	0.10957	0.11669	0.490	0.19314	0.33154	0.38712	0.43865	0.48818
0.245	0.07353	0.09730	0.10597	0.11386	0.12139	0.495	0.19578	0.33775	0.39486	0.44781	0.49871
0.250	0.07571	0.10075	0.10991	0.11824	0.12620	0.500	0.19843	0.34402	0.40267	0.45708	0.50937
0.255	0.07791	0.10425	0.11391	0.12271	0.13111	0.510	0.20373	0.35674	0.41857	0.47594	0.53110
0.260	0.08012	0.10781	0.11799	0.12726	0.13612	0.520	0.20905	0.36970	0.43480	0.49524	0.55336
0.265	0.08234	0.11142	0.12213	0.13190	0.14124	0.530	0.21440	0.38291	0.45138	0.51499	0.57616
0.270	0.08458	0.11508	0.12635	0.13663	0.14646	0.540	0.21976	0.39635	0.46831	0.53519	0.59952
0.275	0.08683	0.11880	0.13064	0.14145	0.15178	0.550	0.22514	0.41004	0.48559	0.55584	0.62342
0.280	0.08909	0.12257	0.13500	0.14635	0.15721	0.560	0.23055	0.42397	0.50322	0.57694	0.64788
0.285	0.09137	0.12640	0.13944	0.15135	0.16274	0.570	0.23597	0.43815	0.52120	0.59850	0.67289
0.290	0.09366	0.13028	0.14395	0.15643	0.16838	0.580	0.24140	0.45257	0.53955	0.62053	0.69848
0.295	0.09596	0.13422	0.14853	0.16161	0.17413	0.590	0.24686	0.46724	0.55825	0.64302	0.72463
0.300	0.09828	0.13822	0.15318	0.16687	0.17998	0.600	0.25233	0.48216	0.57731	0.66599	0.75136
0.305	0.10060	0.14226	0.15791	0.17223	0.18594	0.610	0.25782	0.49733	0.59674	0.68943	0.77867
0.310	0.10294	0.14637	0.16271	0.17768	0.19201	0.620	0.26332	0.51275	0.61654	0.71334	0.80657
0.315	0.10529	0.15052	0.16759	0.18322	0.19819	0.630	0.26884	0.52843	0.63670	0.73773	0.83505
0.320	0.10765	0.15474	0.17254	0.18885	0.20448	0.640	0.27437	0.54436	0.65724	0.76261	0.86412
0.325	0.11002	0.15901	0.17756	0.19458	0.21088	0.650	0.27992	0.56054	0.67815	0.78798	0.89380
0.330	0.11240	0.16333	0.18266	0.20040	0.21739	0.660	0.28548	0.57698	0.69943	0.81384	0.92408
0.335	0.11480	0.16771	0.18784	0.20631	0.22401	0.670	0.29106	0.59367	0.72110	0.84019	0.95496
0.340	0.11720	0.17214	0.19309	0.21232	0.23074	0.680	0.29665	0.61063	0.74314	0.86704	0.98646
0.345	0.11961	0.17663	0.19842	0.21842	0.23759	0.690	0.30225	0.62785	0.76557	0.89439	1.01857
0.350	0.12203	0.18118	0.20382	0.22462	0.24455	0.700	0.30786	0.64532	0.78839	0.92225	1.05131
0.350	0.122034	0.181179	0.203818	0.224617	0.244552	0.710	0.31349	0.66306	0.81159	0.95061	1.08467
0.355	0.12447	0.18578	0.20930	0.23091	0.25163	0.720	0.31913	0.68107	0.83518	0.97949	1.11866
0.360	0.12691	0.19044	0.21485	0.2373	0.25882	0.730	0.32477	0.69933	0.85917	1.00888	1.15328
0.365	0.12936	0.19515	0.22048	0.24378	0.26612	0.740	0.33043	0.71787	0.88355	1.03879	1.18855
0.370	0.13182	0.19992	0.22619	0.25037	0.27355	0.750	0.33611	0.73667	0.90832	1.06923	1.22446
0.375	0.13428	0.20475	0.23198	0.25705	0.28108	0.760	0.34179	0.75574	0.93350	1.10019	1.26101
0.380	0.13676	0.20963	0.23784	0.26382	0.28874	0.770	0.34748	0.77508	0.95908	1.13167	1.29822
0.385	0.13925	0.21457	0.24379	0.2707	0.29652	0.780	0.35318	0.79469	0.98506	1.16369	1.33609
0.390	0.14174	0.21956	0.24981	0.27768	0.30441	0.790	0.35889	0.81458	1.01145	1.19625	1.37462
0.395	0.14424	0.22462	0.25591	0.28475	0.31242	0.800	0.36461	0.83474	1.03825	1.22934	1.41381
0.400	0.14675	0.22972	0.26209	0.29192	0.32056						
0.405	0.14927	0.23489	0.26834	0.29920	0.32881	0.810	0.37035	0.85517	1.06546	1.26298	1.45367
0.410	0.15180	0.24011	0.27468	0.30657	0.33718	0.820	0.37609	0.87588	1.09308	1.29716	1.49421
0.415	0.15433	0.24539	0.28110	0.31405	0.34568	0.830	0.38183	0.89686	1.12112	1.33190	1.53543
0.420	0.15687	0.25073	0.28759	0.32163	0.35430	0.840	0.38759	0.91813	1.14958	1.36718	1.57733
0.425	0.15942	0.25612	0.29417	0.32931	0.36304	0.850	0.39336	0.93967	1.17846	1.40302	1.61992
0.430	0.16197	0.26158	0.30083	0.33709	0.37191	0.860	0.39913	0.96150	1.20776	1.43942	1.66320
0.435	0.16453	0.26709	0.30757	0.34498	0.38090	0.870	0.40492	0.98361	1.23748	1.47638	1.70718
0.440	0.16710	0.27265	0.31439	0.35297	0.39001	0.880	0.41071	1.00600	1.26763	1.51391	1.75186
0.445	0.16968	0.27828	0.32129	0.36106	0.39925	0.890	0.41650	1.02868	1.29821	1.55201	1.79725
0.450	0.17226	0.28396	0.32827	0.36926	0.40862	0.900	0.42231	1.05164	1.32923	1.59067	1.84334

Table 3A-1 *(Continued)*

η_0	Value of ϕ					η_0	Value of ϕ				
	$m = 0$	$m = 1.0$	$m = 1.5$	$m = 2.0$	$m = 2.5$		$m = 0$	$m = 1.0$	$m = 1.5$	$m = 2.0$	$m = 2.5$
0.910	0.42812	1.07489	1.36067	1.62992	1.89015	1.31	0.66524	2.25468	2.99960	3.70806	4.39526
0.920	0.43394	1.09843	1.39256	1.66974	1.93767	1.32	0.67126	2.29076	3.05072	3.77362	4.47486
0.930	0.43977	1.12226	1.42488	1.71015	1.98592	1.33	0.67728	2.32719	3.10236	3.83988	4.55536
0.940	0.44561	1.14638	1.45764	1.75114	2.03489	1.34	0.68330	2.36395	3.15453	3.90684	4.63673
0.950	0.45145	1.17080	1.49084	1.79271	2.08460	1.35	0.68932	2.40106	3.20723	3.97452	4.71900
0.960	0.45730	1.19550	1.52449	1.83488	2.13503	1.36	0.69535	2.43850	3.26046	4.04292	4.80217
0.970	0.46315	1.22050	1.55859	1.87765	2.18621	1.37	0.70138	2.47629	3.31422	4.11203	4.88623
0.980	0.46901	1.24580	1.59314	1.92101	2.23813	1.38	0.70741	2.51442	3.36851	4.18186	4.97120
0.990	0.47488	1.27140	1.62814	1.96498	2.29080	1.39	0.71345	2.55290	3.42335	4.25242	5.05707
1.000	0.48075	1.29729	1.66359	2.00954	2.34422	1.40	0.71949	2.59173	3.47872	4.32370	5.14385
1.010	0.48663	1.32348	1.69950	2.05472	2.39840	1.41	0.72553	2.63090	3.53463	4.39571	5.23155
0.020	0.49251	1.34997	1.73586	2.10051	2.45333	1.42	0.73158	2.67042	3.59109	4.46845	5.32016
1.030	0.49840	1.37677	1.77269	2.14691	2.50903	1.43	0.73762	2.71029	3.64809	4.54193	5.40970
1.040	0.50430	1.40387	1.80998	2.19393	2.56550	1.44	0.74367	2.75052	3.70563	4.61615	5.50016
1.050	0.51020	1.43127	1.84773	2.24157	2.62274	1.45	0.74973	2.79109	3.76373	4.69111	5.59155
1.060	0.51611	1.45898	1.88596	2.28983	2.68076	1.46	0.75578	2.83202	3.82237	4.76681	5.68387
1.070	0.52202	1.48700	1.92465	2.33872	2.73955	1.47	0.76184	2.87330	3.88157	4.84326	5.77713
1.080	0.52794	1.51533	1.96381	2.38823	2.79913	1.48	0.76790	2.91494	3.94133	4.92045	5.87133
1.090	0.53386	1.54396	2.00345	2.43839	2.85950	1.49	0.77397	2.95694	4.00164	4.99841	5.96647
1.100	0.53979	1.57291	2.04356	2.48917	2.92067	1.50	0.78003	2.99929	4.06251	5.07711	6.06256
1.110	0.54572	1.60216	2.08415	2.54060	2.98262	1.51	0.78610	3.04200	4.12394	5.15657	6.15960
1.120	0.55165	1.63173	2.12522	2.59267	3.04538	1.52	0.79217	3.08508	4.18594	5.23680	6.25760
1.130	0.55760	1.66162	2.16677	2.64538	3.10895	1.53	0.79824	3.12851	4.24850	5.31779	6.35655
1.140	0.56354	1.69182	2.20881	2.69874	3.17332	1.54	0.80432	3.17231	4.31163	5.39954	6.45647
1.150	0.56949	1.72234	2.25133	2.75276	3.23851	1.55	0.81040	3.21647	4.37532	5.48207	6.55736
1.160	0.57545	1.75317	2.29434	2.80743	3.30451	1.56	0.81647	3.26100	4.43959	5.56536	6.65921
1.170	0.58141	1.78433	2.33784	2.86275	3.37133	1.57	0.82256	3.30589	4.50443	5.64944	6.76204
1.180	0.58737	1.81580	2.38184	2.91874	3.43898	1.58	0.82864	3.35115	4.56984	5.73429	6.86584
1.190	0.59334	1.84760	2.42633	2.97539	3.50746	1.59	0.83473	3.39678	4.63584	5.81992	6.97063
1.200	0.59931	1.87972	2.47132	3.03271	3.57677	1.60	0.84081	3.44278	4.70241	5.90633	7.07640
1.21	0.60528	1.91216	2.51681	3.09069	3.64692	1.61	0.84691	3.48914	4.76956	5.99354	7.18316
1.22	0.61126	1.94493	2.56279	3.14935	3.71790	1.62	0.85300	3.53588	4.83729	6.08153	7.29091
1.23	0.61725	1.97802	2.60929	3.20869	3.78973	1.63	0.85909	3.58300	4.90561	6.17031	7.39965
1.24	0.62323	2.01145	2.65628	3.26870	3.86241	1.64	0.86519	3.63048	4.97452	6.25989	7.50940
1.25	0.62922	2.04520	2.70379	3.32940	3.93595	1.65	0.87129	3.67834	5.04402	6.35026	7.62015
1.26	0.63522	2.07928	2.75181	3.39078	4.01033	1.66	0.87739	3.72658	5.11410	6.44144	7.73190
1.27	0.64122	2.11369	2.80033	3.45285	4.08558	1.67	0.88349	3.77520	5.18478	6.53342	7.84466
1.28	0.64722	2.14844	2.84937	3.51561	4.16170	1.68	0.88959	3.82419	5.25605	6.62620	7.95844
1.29	0.65322	2.18351	2.89893	3.57906	4.23868	1.69	0.89570	3.87357	5.32792	6.71980	8.07323
1.30	0.65923	2.21893	2.94901	3.64321	4.31653	1.70	0.90181	3.92332	5.40039	6.81420	8.18905